

Research article

Stock Investment Decision in Nigeria; A PC Approach

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Abstract

Growth index of two groups of stocks listed on the floor of the Nigerian Stock Exchange were compared using Principal Component Analysis Approach, with the aim of reducing the dimensionality of the data and to test whether the Food, Beverage and Brewery Industry had a better growth return than the Financial Institution. Data were sourced from the Nigerian Stock Exchange daily and weekly reports spanning over a period of 100 successive weeks. The result of the analysis showed that the Financial Institution had a better growth return than the Food, Beverage and Brewery Industry. **Copyright © WJMCR, all rights reserved.**

Keywords: Stock Investment, PCA, Financial Institution, Nigerian Stock Exchange

1. Introduction

A stock is a share in the ownership of a company. Stock represents a claim on the company's assets and earnings. As you acquire more stock, your ownership stake in the company becomes greater. Holding a company's stock means that you are one of the many owners (shareholders) of a company and, as such, you have a claim to everything the company owns. As an owner, you are entitled to your share of the company's earnings

as well as any voting rights attached to the stock. A stock is represented by a stock certificate usually kept electronically by the brokerage. The importance of stock ownership is your claim on assets and earnings, without this, the stock wouldn't be worth the paper its printed on. Another extremely important features of stock is its limited liability, which means that, as an owner of a stock, you are not personally liable if the company is not able to pay its debts. Other company such as partnerships are setup so that if the partnership goes bankrupt, the creditors can come after the partners (shareholders) personally and sell off their house, car, furniture e.t.c. Owning stocks means that, no matter what, the maximum value you can loose is the value of your investment. Even if a company of which you are a shareholder goes bankrupt, you can never loose your personal assets. Thus, the objectives of this paper are;

1. To perform the principal component analysis of hundred successive weeks growth indices of the 5 selected Food, Beverage and Brewery Firms and 5 selected companies from the Financial Sector.
2. To test the significance of the difference in the entire eigenvector between the two sectors.
3. To determine which of the sectors has a greater patronage in the stock market within the hundred successive weeks.

2. Methodology

To achieve the set objectives, data pertaining the subject matter was obtained from the daily and weekly report of Nigerian Stock Exchange, it was for a 3 year period (2013-2015 financial year). These were actually for hundred successive weeks. The target population for this study is all stock traded at the floor of Nigerian Stock Exchange. From all the stocks, ten stocks were selected, five each from the Financial Institution and the Food, Beverage and Brewery Industry. The data was extracted on the bases of the Current Friday Closing Price (C.F.C.P) and Previous Friday Closing Price (P.F.C.P). Then the weekly rate of return is;

$$W.R.R = \frac{C.F.C.P - P.F.C.P}{P.F.C.P}$$

2.1 Principal Component Analysis

Principal component analysis is a multivariate technique for transforming a set of related (correlated) variables into a set of unrelated (uncorrelated) variables that account for decreasing proportions of the variation of the original observations (Rencher, 2002). The rationale behind the method is an attempt to reduce the complexity of the data by decreasing the number of variables that need to be considered. If the first few of the derived variables (the principal components) among them account for a large proportion of the total variance of the observed variables, they can be used to provide a convenient summary of the data and to simplify subsequent analysis. Algebraically, principal component are particular linear combinations of the p random variables X_1, X_2, \dots, X_p . Geometrically, these linear combination represents the selection of new coordinate system obtained by rotating the original system with X_1, X_2, \dots, X_p as the coordinate axes. The new axes represents the directions with maximum variability and provide a simpler and more parsimonious description of the covariance structure. Principal components depend solely on the covariance matrix Σ (or the correlation matrix ρ) of X_1, X_2, \dots, X_p . Their development does not require a multivariate normal assumption. let the random vector $X^T =$

$[X_1, X_2, \dots, X_p]$ have the covariance matrix Σ with eigenvalue $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p \geq 0$. Consider the linear combination

$$\begin{aligned} Y_1 &= \mathbf{a}'_1 \mathbf{X} = a_{11}X_1 + a_{12}X_2 + \dots + a_{1p}X_p \\ Y_2 &= \mathbf{a}'_2 \mathbf{X} = a_{21}X_1 + a_{22}X_2 + \dots + a_{2p}X_p \\ &\vdots \\ Y_p &= \mathbf{a}'_p \mathbf{X} = a_{p1}X_1 + a_{p2}X_2 + \dots + a_{pp}X_p \end{aligned}$$

Then,

$$\text{Var}(Y_i) = \mathbf{a}'_i \Sigma \mathbf{a}_i \quad i = 1, 2, \dots, p$$

$$\text{Cov}(Y_i, Y_k) = \mathbf{a}'_i \Sigma \mathbf{a}_k \quad i, k = 1, 2, \dots, p$$

Note;

First principle component = linear combination $\mathbf{a}'_1 \mathbf{X}$ that maximizes $\text{Var}(\mathbf{a}'_1 \mathbf{X})$ subject to $\mathbf{a}'_1 \mathbf{a}_1 = 1$

Second principle component = linear combination $\mathbf{a}'_2 \mathbf{X}$ that maximizes $\text{Var}(\mathbf{a}'_2 \mathbf{X})$

subject to $\mathbf{a}'_2 \mathbf{a}_2 = 1$ and $\text{Cov}(\mathbf{a}'_1 \mathbf{X}, \mathbf{a}'_2 \mathbf{X}) = 0$

At the i th step,

i th principle component = linear combination $\mathbf{a}'_i \mathbf{X}$ that maximizes $\text{Var}(\mathbf{a}'_i \mathbf{X})$

subject to $\mathbf{a}'_i \mathbf{a}_i = 1$ and $\text{Cov}(\mathbf{a}'_i \mathbf{X}, \mathbf{a}'_k \mathbf{X}) = 0$ for $k < i$.

Consider the covariance matrix of a Bivariate data

$$\Sigma = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{11} & \delta_{12} \end{pmatrix}$$

and the derived correlation matrix

$$\rho = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}$$

The proportion of the total variance explained by the first principal component is

$$\Psi_{X_1} = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

which is larger than that explained by the second principal component when the X's are not standardized.

In determining the number of Principal Components to retain, the amount of total variance explained, the relative sizes of the eigenvalues, { Jolliffe (2002) } suggests using a cutoff on the eigenvalue of 0.7 when correlation matrices are analyzed., and a visual inspection of the scree plots are of prior importance.

3. Data Analysis and Result

Table 1: Covariance Matrix; Food, Beverage and Brewery Industry.

0.004129	0.001074	0.0003845	-0.000461	0.0006331
0.001074	0.002986	0.0005163	-0.00107	-0.0002615
0.0003845	0.0005163	0.001689	-0.000242	-0.0001065
-0.0004614	-0.00107	-0.000242	0.007827	0.001891
0.00006334	-0.000261	-0.000106	0.001891	0.005649

Table 2: Initial Eigenvalues; Food, Beverage and Brewery Industry.

Component	Total	% of Variance	Cumulative %
1	0.009136	41.008	41.008
2	0.005495	24.664	65.672
3	0.003858	17.317	82.989
4	0.002285	10.255	93.245
5	0.001505	6.755	100.000

Table 3: Eigenvectors; Food, Beverage and Brewery Industry.

-0.094	0.758	0.506	-0.401	-0.005
-0.118	0.194	0.239	0.322	0.888
0.929	-0.184	0.319	0.040	0.012
0.598	0.623	-0.498	0.083	0.008
-0.322	0.449	0.412	0.684	-0.240

Table 4: Principal Components; Food, Beverage and Brewery Industry.

	1	2	3	4	5
	0.094	-0.118	0.929	0.598	-0.322
	0.758	0.194	-0.184	0.623	0.449
	0.506	0.239	0.139	-0.498	0.412
	-0.401	0.322	0.040	0.083	0.684
	-0.005	0.888	-0.012	0.008	-0.240

Putting the variables, we have;

$$Y_1 = 0.094 X_1 + 0.758 X_2 + 0.506X_3 - 0.401X_4 - 0.005X_5$$

$$Y_2 = -0.118 X_1 + 0.194 X_2 + 0.239X_3 + 0.322X_4 + 0.888X_5$$

$$Y_3 = 0.929 X_1 - 0.184 X_2 + 0.139X_3 + 0.040X_4 - 0.012X_5$$

$$Y_4 = 0.598 X_1 + 0.623 X_2 - 0.498X_3 + 0.083X_4 + 0.008X_5$$

$$Y_5 = -0.322 X_1 + 0.449X_2 + 0.412X_3 + 0.684X_4 - 0.240X_5$$

Table 5: Covariance Matrix; Financial Institution

0.001944	0.0008634	0.0003128	0.0002814	0.0004347
0.0008634	0.005620	0.0009584	0.0008893	0.0009476
0.0003128	0.0009583	0.007676	-0.0003192	0.0001304
0.0002814	0.0008893	-0.000319	0.003724	-0.0000403
0.0004347	0.0009476	-0.0001304	-0.0004034	0.0002262

Table 6: Initial Eigenvalues; Financial Institution.

Component	Total	% of Variance	Cumulative %
1	0.008168	38.479	38.479
2	0.005898	28.214	66.693
3	0.003389	15.967	82.659
4	0.002081	9.804	92.464
5	0.001600	7.536	100.000

Table 7: Eigenvectors; Financial Institution

0.227	0.319	-0.062	0.504	0.767
0.505	0.803	-0.223	-0.225	-0.013
0.924	-0.370	0.095	0.014	-0.016
0.038	0.486	0.863	0.105	-0.076
0.180	0.326	-0.337	0.746	-0.437

Table 8: Principal Components; Financial Institution

	1	2	3	4	5
0.227	0.505	0.924	0.038	0.180	
0.319	0.803	-0.370	0.486	0.326	
-0.062	-0.223	0.095	0.863	-0.337	
0.504	-0.225	0.014	0.105	0.746	
0.767	-0.013	-0.016	-0.076	-0.437	

Putting the variables, we have;

$$Y_1 = 0.227 X_1 + 0.319 X_2 + 0.062X_3 + 0.504X_4 + 0.767X_5$$

$$Y_2 = 0.505 X_1 + 0.803 X_2 - 0.223X_3 - 0.225X_4 - 0.103X_5$$

$$Y_3 = 0.924 X_1 - 0.370 X_2 + 0.095X_3 + 0.014X_4 - 0.016X_5$$

$$Y_4 = 0.038 X_1 + 0.486 X_2 + 0.863X_3 + 0.105X_4 - 0.076X_5$$

$$Y_5 = 0.180 X_1 + 0.326X_2 - 0.337X_3 + 0.764X_4 - 0.437X_5$$

3.1 Test of Significance

$$H_0 : e_{ij} = 0$$

$$H_1 : e_{ij} \neq 0$$

$$\chi^2 = n(e_i C_i^T S^{-1} C_i + \frac{1}{e_i} - C_i^T S C_i - 2)$$

Aim: To test whether the two- covariance matrices are homogenous with respect to the growth.

0.004129	0.001074	0.0003845	-0.000461	0.0006331
0.001074	0.002986	0.0005163	-0.00107	-0.0002615
0.0003845	0.0005163	0.001689	-0.000242	-0.0001065

$$S_1 = \begin{vmatrix} -0.0004614 & -0.00107 & -0.000242 & 0.007827 & 0.001891 \\ 0.00006334 & -0.000261 & -0.000106 & 0.001891 & 0.005649 \end{vmatrix}$$

$$S_2 = \begin{vmatrix} 0.001944 & 0.0008634 & 0.0003128 & 0.0002814 & 0.0004347 \\ 0.0008634 & 0.005620 & 0.0009584 & 0.0008893 & 0.0009476 \\ 0.0003128 & 0.0009583 & 0.007676 & -0.0003192 & 0.0001304 \\ 0.0002814 & 0.0008893 & -0.000319 & 0.003724 & -0.0000403 \\ 0.0004347 & 0.0009476 & -0.0001304 & -0.0004034 & 0.0002262 \end{vmatrix}$$

$$P_1 = \begin{vmatrix} 0.094 & -0.118 & 0.929 & 0.598 & -0.322 \\ 0.758 & 0.194 & -0.184 & 0.623 & 0.449 \\ 0.506 & 0.239 & 0.139 & -0.498 & 0.412 \\ -0.401 & 0.322 & 0.040 & 0.083 & 0.684 \\ -0.005 & 0.888 & -0.012 & 0.008 & -0.240 \end{vmatrix}$$

$$P_2 = \begin{vmatrix} 0.227 & 0.505 & 0.924 & 0.038 & 0.180 \\ 0.319 & 0.803 & -0.370 & 0.486 & 0.326 \\ -0.062 & -0.223 & 0.095 & 0.863 & -0.337 \\ 0.504 & -0.225 & 0.014 & 0.105 & 0.746 \\ 0.767 & -0.013 & -0.016 & -0.076 & -0.437 \end{vmatrix}$$

Where S_1, S_2, P_1, P_2 are the sample covariance matrices and their eigenvalues and vectors for the respective sample stocks sectors.

Also, $e_{11} = 0.009136$, $e_{21} = 0.005495$, $e_{31} = 0.003858$, $e_{41} = 0.002285$, $e_{51} = 0.001505$ and, $e_{12} = 0.008168$, $e_{22} = 0.005989$, $e_{32} = 0.003389$, $e_{42} = 0.002081$, $e_{52} = 0.001600$. where C_i is the rows of the second sample covariance and e_i is the eigenvalues of the Food, Beverage and Brewery Industry. Therefore,

$$\begin{vmatrix} 277.11616 & -92.646637 & -35.52725 & 12.250067 & -40.116457 \\ -92.646712 & 401.22934 & -94.5443 & 43.416679 & 12.6405023 \\ -35.536921 & -94.54203 & 630.00134 & 1.8351923 & 10.8692649 \end{vmatrix}$$

$$S^{-1} = \begin{vmatrix} 12.2622052 & 43.418735 & 1.8462695 & 146.13449 & -48.247991 \\ -40.10936 & 12.612703 & 10.816975 & -48.25094 & 198.457425 \end{vmatrix}$$

From here, we can deduce that,

$$C_1 = \begin{vmatrix} 0.001944 & 0.0008634 & 0.0003128 & 0.0002814 & 0.0004347 \end{vmatrix}$$

$$\begin{vmatrix} 0.001944 \\ 0.0008634 \\ 0.0003128 \\ 0.0002814 \\ 0.0004347 \end{vmatrix}$$

$$C_1^T =$$

$$e_{11} = 0.009136$$

$$C_1 S^{-1} C_1^T = 0.001019$$

$$e_{11} C_1 S^{-1} C_1^T = 0.00000931$$

$$C_1 S_1 C_1^T = -0.0000000243$$

$$1/e_{11} = 109.4571$$

$$\chi^2 = 10745.1$$

Also,

$$C_2 = \begin{vmatrix} 0.0008634 & 0.005620 & 0.0009584 & 0.0008893 & 0.0009476 \end{vmatrix}$$

$$\begin{vmatrix} 0.0008634 \\ 0.005620 \\ 0.0009583 \\ 0.0008893 \\ 0.0009476 \end{vmatrix}$$

$$C_2^T =$$

$$e_{21} = 0.005495$$

$$C_2 S_2^{-1} C_2^T = 0.012238$$

$$e_{21} C_2 S_2^{-1} C_2^T = 0.0000673$$

$$C_2 S_2 C_2^T = 0.000000116$$

$$1/e_{21} = 181.9836$$

$$\chi^2 = 17998.37$$

Similarly,

$$C_3 = \begin{vmatrix} 0.0003128 & 0.0009583 & 0.007676 & -0.0003192 & 0.0001304 \\ 0.0003128 & 0.0009584 & 0.007676 & -0.000319 & -0.0001304 \end{vmatrix}$$

$$C_3^T =$$

$$e_{31} = 0.003858$$

$$C_3 S_3^{-1} C_3^T = 0.035905$$

$$e_{31} C_3 S_3^{-1} C_3^T = 0.000139$$

$$C_3 S_3 C_3^T = 0.000000115$$

$$1/e_{31} = 259.2017$$

$$\chi^2 = 25720.18$$

And,

$$C_4 = \begin{vmatrix} 0.0002814 & 0.0008893 & -0.000319 & 0.003724 & -0.0000403 \\ 0.0002814 & 0.0008893 & -0.0003192 & 0.003724 \end{vmatrix}$$

$$\left| -0.0004034 \right|$$

$$C_4^T =$$

$$e_{41} = 0.002285$$

$$C_4 S_4^{-1} C_4^T = 0.002768$$

$$e_{41} C_4 S_4^{-1} C_4^T = 0.00000632$$

$$C_4 S_4 C_4^T = 0.000000104$$

$$1/e_{41} = 437.6368$$

$$\chi^2 = 43563.68$$

Finally,

$$C_5 = \left| \begin{array}{ccccc} 0.0004347 & 0.0009476 & -0.0001304 & -0.0004034 & 0.0002262 \end{array} \right|$$

$$\left| \begin{array}{c} 0.0004347 \\ 0.0009476 \\ 0.0001304 \\ -0.0004034 \\ 0.0002262 \end{array} \right|$$

$$C_5^T =$$

$$e_{51} = 0.1505$$

$$C_5 S_5^{-1} C_5^T = 0.001322$$

$$e_{51} C_5 S_5^{-1} C_5^T = 0.00000199$$

$$C_5 S_5 C_5^T = 0.0000000333$$

$$1/e_{51} = 664.4518$$

$$\chi^2 = 66245.18$$

By comparing the theoretical Chi-squared value with P-1 degrees of freedom for all cases, indicates the rejection of H_0 , so the stocks are heterogeneous with respect to their growth index.

4. Conclusion and Recommendation.

From the analysis thus far, we hereby conclude that the two groups of stocks are not homogenous rather they are heterogeneous with respect to their growth index. The stock returns from the Financial Institution had a better growth index than the stock returns of the Food, Beverage and Brewery Industry, this can be attributed to the

daily/weekly high market activity of the Financial Institution, while in the case of Food, Beverage and Brewery Industry, their performance could be due to the non-uniform industry activity. We therefore recommend that investors invest in stock of the Financial Institution for better return of investment.

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